James B. Nation, "Assessment Plan," Department of Mathematics, College of Natural Sciences (Presented by William Lampe)

Desired learning outcomes of mathematics majors were determined by the department's Assessment Committee. Three measures for assessment of the major were selected: a capstone seminar, an assessment exam, and an alumni survey. The capstone seminar, Math 480, was offered for the first time in Spring 2002. It is a one-credit seminar course in which each student is required to write a short paper on a mathematical topic and present it. The assessment exam consists of two parts. The first six questions concern calculus, linear algebra, and differential equations. The second part consists of two questions from each of the six 400-level mathematics sequences. The exam was constructed by soliciting questions from faculty members currently or recently teaching 400-level courses. The exam was given to students enrolled in Math 480 and to students projected for graduation in Spring 2002 who were not enrolled in Math 480. The alumni survey is an ongoing effort to determine the alumni's satisfaction with and recommendations for the program.

ASSESSMENT FOR THE MATHEMATICS MAJOR:

REPORT FOR 2001-2002

A copy of the Mathematics Department's Assessment Plan is attached. This report provides a summary of the department's assessment activities during the academic year 2001-2002. First we state the goals of the mathematics major from the Assessment

Plan.

1. Goals of the Program

Recipients of an undergraduate degree in mathematics are expected to understand and apply:

- basic real analysis of one variable;
- calculus of several variables and vector analysis;
- basic linear algebra and theory of vector spaces;
- concerted study of at least one advanced topic of mathematics, chosen from: advanced calculus, abstract algebra, set theory and logic, probability and statistics, partial differential equations, and numerical analysis.

In addition, they are expected to acquire the ability and skills to:

- give direct proofs, proofs by contradiction, and proofs by induction;
- formulate definitions and give examples and counterexamples;
- read mathematics without supervision;
- follow and explain algorithms; and

- apply mathematics to other fields

2. Assessment Activities for 2001-2002

In accordance with the plan, the Mathematics Department offered a one-credit capstone seminar, Math 480, in the Spring semester of 2002. The course has not yet been made a major requirement by the department. The course was taught by Professor Adolf Mader. There were three students enrolled, two graduating seniors and one unclassified graduate student who had graduated in December 2001. Specific skills taught included the use of MAPLE, and LaTEX. Each of these students completed and presented a

senior report or project and lectured in class. These reports were of high quality. A more detailed report on the course is attached. The faculty devised an assessment exam covering the entire mathematics major. About half a dozen faculty members contributed questions. Four students took the exam, the two seniors from Math 480 and two volunteers. These were above-average mathematics majors.

The exam had questions from the elementary courses as well as the advanced ones. In general, each question had several parts, of increasing difficulty. The students indicated which advanced courses each had taken, and were expected only to answer questions from those courses (and the elementary ones).

The results on the elementary part of the exam were mixed but generally disappointing. Some questions were answered correctly, but equally easy questions were omitted. The results were somewhat better in the advanced courses taken more recently, but still not exceptional. One of the questions from one advanced course concerned a chapter skipped by this year's instructor. The students were not expected to prepare for the exam and evidently did not. The Mathematics Department sends questionnaires to all mathematics majors upon graduation. Eleven questionnaires were sent out, and four were returned. The paucity of returns makes it difficult to draw any conclusions from this part of the Assessment Program. A summary of the results is attached as well as a copy of the questionnaire.

3. Proposed Changes

The department needs to find a way to reinforce basic concepts, especially in calculus and linear algebra. This could be done in the senior seminar or through a separate 300-level course. A second semester to Math 321, to be known as Math 322 and required for the major, has been proposed by some faculty and is under consideration. At the next department meeting, the faculty will consider making the capstone seminar a requirement of the program; however, lack of resources imperils this idea. Modifying the capstone seminar to emphasize the synthesis of the material learned in other courses will also be discussed by the department. Consideration will also be given to making the assessment exam mandatory so that the data derived is more reflective of the general impact of the major.

MATH 480 Spring 2002

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The course began with an introduction to LaTex. The students were asked to download free programs and they were made available on Department machines. They were given sample manuscripts, both the Latex documents and the output generated. They were asked to attend a talk in the Undergraduate Seminar and write a report using Latex.

One hour was spent in discussing the AMS Subject Classification system.

This was followed by an introduction to Maple. The students were introduced to the theory of integral matrices which served as source for programming problems. The students did program in Maple eventually. They went through some tutorials available in Maple and reported on them in class.

The later part of the course revolved around term papers on topics chosen

by the students. They did submit outlines, presented in class preliminary reports, and a final report. I read the final reports and met with the students individually to discuss formulations and designs for mathematical exposition. A revised final report was supposed to be returned to me in triplicate but only one of the three students did so.

The students acquired basic skills in Latex and Maple and demonstrated that they could use these programs. Each student wrote a very nice paper, and each chose a different topic. The topics were The Knapsack Problem and Encryption, Fast Fourier Transforms, and Models of Computation.

One of the students gave a well-prepared talk in the Undergraduate Seminar Series of the Department. In the process he presented computer simulations of Turing machines.

Evaluation. All three students acquired important skills in mathematical type setting and use of computer algebra systems. They had to write and they all wrote well. I did not do any measurable testing, but I don't hesitate to assert in all three cases high ability and good personal attitudes.

RESULTS OF THE SPRING 2002 MATH ASSESSMENT EXAM

Four graduating math majors took the mathematics assessment exam in May 2002. The exam was compiled by the associate chair from questions submitted by half a dozen faculty members. This report will summarize the results of the exam in the various areas tested.

Calculus. The routine tangent line and integration problems were solved correctly by most students. Only one student could find the Taylor series for sin 2x about x = 0. The

same student was the only one to find the closed form of a sequence given recursively. Most students could find a scalar field given its gradient, but none could use this information to do a line integral. Most could solve a constant coefficient second order initial value problem. The results for this section of the test were thus generally satisfactory, though room remains for improvement.

Linear Algebra. There was one four-part problem concerning the null space of a matrix and the solution of Ax = b. Half the parts were omitted, and half the solutions attempted were incorrect. Clearly this remains a problem area.

Math 402-403. Two students had taken this sequence, and one had taken 402 only. One student solved one of the two questions. The students had not taken the course recently, which may have been a factor.

Math 407-408. Two students had taken this sequence. One got one problem correct, the other one and a half. This seems a reasonable result.

Math 412-413. Two students had taken this sequence. Both correctly solved a problem on factorization over different fields, and could not do a proof using quotient groups.

Math 431-432. The one student who had taken this sequence got one problem correct, and another student answered parts of both questions

Math 454-455. No student had taken this sequence.

Math 471-472. One student had taken this sequence, and two had taken Math 471 only. Two students solved part of the first problem. The second problem was from a section not covered this term.

It is not clear how one should interpret the results of the test, except to note that linear algebra is a notable weakness. It is doubtful that the present test properly measures our students' strengths or weaknesses, and perhaps it could be refined. However, the test does seem to support the idea that a review component of the senior seminar would be valuable.

QUESTIONNAIRE FOR MATHEMATICS GRADUATES:

REPORT FOR 2001-2002

Eleven people graduated with a major in mathematics during 2001-2002. These graduates were sent the standard questionnaire. Four filled them out and sent them back. This is a summary of the responses.

1. Mathematics courses of most value? Of least value?

There was no overlap on the opinion as to most valuable courses or as to least valuable courses. Most valuable courses listed included 431-2, with its requirement to write many, many

proofs, 371, 412-3, 407-8, calculus and differential equations. Only two listed least valuable courses, and these were 190 and 311.

2. Courses not taken that should have been taken

Two graduates listed 302 and 402-3. Also listed were 404, 407-8, 471-2, 351-2, 612, and lattice theory.

3. Courses not offered that should have been

The responses were 475, 480, and vector analysis.

4. Opportunities available as a result of one's mathematics major

The responses were individual and varied. They were

- nice offers from graduate programs
- not much
- algorithm development
- in a developing country, not much except teaching

5. Were WI math courses valuable?

Responses ranged from helpful to most valuable; \carefully scrutinized proofs were most helpful."

6. What one change in the major would you recommend?

One person recommended splitting 431-2 into three pieces so that there was more time for later topics. Another suggested that research or a project be required. A third wanted a more organized curriculum, which seemed to mean that each course should have a predictable content without variation.

7. Other comments

One wrote that one semester each of algebra and analysis should be required and the year sequence should be from one of these and grade inflation should be suppressed. Another wanted career counseling. Another commented on the friendliness and helpfulness of the faculty and staff. The fourth liked the department just as it is and the faculty and being required to take the algebra sequence and the analysis sequence. (The latter is perplexing.)

ASSESSMENT PLAN FOR THE MATHEMATICS MAJOR

The Department of Mathematics offers degree programs leading to the BA and BS degrees in mathematics. A departmental committee was formed to devise a plan to provide ongoing assessment of the mathematics major. The committee solicited the input of the entire department, and the proposal will be submitted to the faculty for

approval. The committee, guided by Heisenberg's principle that one cannot observe something without affecting it, sought to design a plan in which the assessment would be closely tied to academic enrichment.

This report adds details to our original proposal, and compares the proposed program with national guidelines for the mathematics major, and with assessment programs for mathematics at other institutions.

1. Goals of the Program

Students majoring in mathematics at UHM are quite diverse in background, ability, and career objectives. Thus our first task was to identify the desired educational outcomes with respect to both the common core of knowledge and proficiency expected of all math majors, and in regard to each student's special interests. The following statement incorporates both aspects.

Recipients of an undergraduate degree in mathematics are expected to understand and apply:

- basic real analysis of one variable;
- calculus of several variables and vector analysis;
- basic linear algebra and theory of vector spaces;
- concerted study of at least one advanced topic of mathematics, chosen from: advanced calculus, abstract algebra, set theory and logic, probability and statistics, partial differential equations, and numerical analysis.

In addition, they are expected to acquire the ability and skills to:

- give direct proofs, proofs by contradiction, and proofs by induction;
- formulate definitions and give examples and counterexamples;
- read mathematics without supervision;
- follow and explain algorithms; and
- apply mathematics to other fields.

Degree Requirements. For the BA degree in mathematics, students must complete 21 credit hours in mathematics courses numbered above 300, including

- Math 321 (an introduction to understanding and writing formal proofs),
- 3 credit hours in a writing-intensive mathematics course,
- 6 credit hours in a sustained two-course sequence approved by the department.

For the BS degree in mathematics, students must complete 24 credit hours in mathematics courses numbered above 300, and 15 credit hours in additional upper division mathematics courses or appropriate non-introductory courses in the natural or information sciences, including

- Math 321,
- 6 credit hours in writing-intensive mathematics courses,
- 6 credit hours in a sustained two-course sequence approved by the department.

In addition, students in the BS program must demonstrate ability to program scientific problems on a computer by passing one of an approved list of courses.

A tally of the number of mathematics majors and minors in recent years is appended to this report.

The Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America established national guidelines for the mathematics major in its 1991 report [1]. The CUPM Report lists seven components which form the structure of the mathematical sciences major:

A. Calculus (with differential equations),

B. Linear algebra,

C. Probability and statistics,

D. Proof-based courses,

E. An in-depth experience in mathematics,

F. Applications and connections,

G. Track courses, departmental requirements and electives.

All of our mathematics majors take linear algebra, though this requirement has not been formalized. Most mathematics majors at UHM take probability and statistics; it has been suggested that this be added as a requirement for the major. Otherwise, our requirements fit the national guidelines well. The degree requirements also correspond well to the stated departmental requirements.

Other MAA guidelines for the mathematics major, taken from [3], include that the department should provide:

- quality faculty advising,
- undergraduate seminars,
- special meeting rooms for majors,
- undergraduate research opportunities.

Math majors are encouraged to see their adviser at least once per semester (and are required to do so once per year). We have weekly undergraduate seminars and a lounge for math majors. We have had more success with sending our students to

summer research programs elsewhere than with incorporating undergraduates into our research, but we are improving and continue to work on this aspect of the program.

2. Assessment Plans

Assessment practices in mathematics departments at American colleges and universities are described in the MAA book Assessment Practices in Undergraduate Mathematics [2]. The departments described therein use a variety of techniques for assessing the major program:

(1) capstone seminars,

(2) senior projects or presentations,

(3) standardized exams,

(4) departmental exams for assessment,

(5) comprehensive exams,

(6) alumni questionnaires,

(7) portfolios,

(8) focus groups.

Most departments used more than one method. We decided to use a capstone seminar which would include a senior paper and presentation, and a departmentally devised exam, while continuing our use of alumni questionnaires.

Capstone Seminar. The Mathematics Department proposes to introduce a one-credit capstone seminar for senior mathematics majors. This course, Math 480, has been approved by the PCC and will begin in the spring semester of 2002.

Each student in the seminar will be required to write a short paper on a mathematical topic, and to present it to the seminar. This would provide our majors with a research experience not available in most courses. It would also provide faculty representatives with an opportunity to interact with, and evaluate, the graduating seniors as a group. Currently, because the students are specializing in different areas, no faculty member knows more than a few of the graduating seniors. The faculty member responsible for the seminar would then provide a report to the department indicating the strengths and weaknesses of that group of students. This part of the assessment, although subjective, would be valuable to the department.

The capstone seminar would be required for graduation, but would be offered on a credit-no credit basis. To receive credit, a student would have to present an acceptable

paper and take the final examination. The final exam would not affect the student's grade for the course. Rather, it would serve as the department's objective assessment tool.

A complete course description of Math 480, including a sample syllabus, is appended to this report. The articles on Saint Mary's College, Kutztown University and Wabash College in [2] provide detailed accounts of those institutions' experience with capstone courses.

Assessment Examination. The design of the examination is crucial to the effectiveness of the department's evaluation. It would be divided into three parts. The first part would test the student's retention of basic calculus and linear algebra. The second part would test the student's ability to produce and write elementary proofs and examples, that is, to think mathematically. The third part would contain questions from each of the six specializations available in our program, and the student would be asked to answer only questions relevant to the courses he or she has taken (and to indicate on the exam which these are). The exams would be graded, summarized, and the summary results distributed to the faculty

A sample exam is appended to this report. The articles on Franklin College, Ball State University and Wabash College in [2] provide detailed accounts of those institutions' experience with a variety of assessment and comprehensive examinations.

Alumni Questionnaires. The Mathematics Department already sends questionnaires to all mathematics majors upon graduation. A copy of this questionnaire is attached. The response rate has varied, but probably averages close to 50%. Most respondents write favorably of their experience as math majors at UHM. This part of our assessment would be continued, and its results summarized and reported regularly to the faculty. The model program in this area is that of the Department of Statistics at Iowa State University, described in detail in [2].

The faculty member assigned to the capstone seminar would be responsible for the department's assessment activities, reporting directly to the Associate Chair, whose office would keep all the records.

3. Implementation

The capstone seminar, Math 480, will be offered for the first time in the spring semester of 2002. The department will have to pass a motion to make the seminar a requirement for the mathematics major, and this will need to be approved, and then only apply to students who declare their major beginning in spring 2002. We hopefully anticipate that there will be reasonable participation prior to spring 2004, when those students should begin to graduate.

We need to consider whether the seminar is to be offered both fall and spring semesters, or only one of these. The standard estimate is that any assessment program

requires that the faculty member in charge be given at least a one-course teaching reduction per year [2]. If the seminar if offered both semesters, then a one-course reduction per semester would be more appropriate.

In either event, implementation of an assessment program will require that a faculty position be restored to the mathematics department. There are currently no teaching reductions available to offer faculty to administer the assessment program: all faculty are needed to maintain course offerings. In order to maintain our current level of operations, some faculty have voluntarily taught extra credits, in return for future course reductions. Under departmental policy, these promised reductions should be honored in the near future. The department currently owes 5 faculty members a total of 8 course reductions, and half a dozen more faculty have partial credits towards future reductions. Lecturers are not available in Honolulu to staff our courses, even if funds were available. This semester we had funds approved to hire lecturers to teach 5 courses; only 2 qualified candidates were found, who were willing to teach 3 courses.

The Associate Chair will teach Math 480 in the spring semester 2002, in addition to his standard load. The Associate Chair's office is already stretched far beyond its capacity to operate effectively, and cannot maintain long term supervision of the assessment program. Without an additional faculty position, the only way out of this quandary would be to teach more courses in large sections-directly contrary to one of the strongest recommendations of the MAA Guidelines [3]. The effect of these considerations is that there is currently no faculty available to operate the assessment program without harming the program itself, though apparently this is what will have to be done.

We are also asked to describe \the program's plan to use the data derived from these measures to make any indicated adjustments in the program (pedagogy, curriculum, advising, etc.)" The Mathematics Department is continually engaged in the process of reviewing and revising its courses and requirements, including the major program. Significant changes in recent years include the requirements for writing-intensive courses, the two-course sequence, and Math 321 for the major. The overwhelming testimony of [2] is that departments with a formal assessment program are engaged in similar refinement processes.

References:

[1] Committee on the Undergraduate Program in Mathematics, The Undergraduate Major in the Mathematical Sciences, Mathematical Association of America, 1991.

[2] B. Gold, S. Keith, W. Marion, eds., Assessment Practices in Undergraduate Mathematics, Mathematical Association of America, 1999.

[3] Task Force to Review the 1993 Guidelines, Guidelines for Programs and Departments in Undergraduate Mathematical Sciences, Revised edition, Mathematical Association of America, 2000.